

## Analytic Representation of Upper Atmosphere Densities Based on Jacchia's Static Diffusion Models

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In a recent paper, Jacchia (1964) has presented a set of models of the upper atmosphere designed to reproduce the satellite drag density data while requiring the minimum amount of tabulation. He uses an empirical expression for the temperature profile,

$$T(z) = T_{\infty} - (T_{\infty} - T_{120}) \exp[-s(z-120)], \quad (1)$$

where  $T_{120}$  is the temperature at 120 km,  $T_{\infty}$  is the exospheric temperature,  $z$  is the altitude in kilometers, and  $s$  is an analytic function of  $T_{\infty}$ . Jacchia presents analytic expressions for  $T_{\infty}$  which represent the solar cycle, short period solar activity, semiannual, diurnal, and geomagnetic activity variations of atmospheric density. The expressions for  $T_{\infty}$  and  $s$  have been determined so that the temperature profile (1) yields atmospheric densities in agreement with satellite drag data (Jacchia and Slowey, 1963) when the diffusive equilibrium equations for  $N_2$ ,  $O_2$ ,  $O$  and  $He$  are integrated numerically using a single set of boundary conditions.

I wish to call attention to a minor modification of Jacchia's procedure which eliminates the numerical integration of the diffusive equilibrium equations, yielding a completely analytic representation of upper atmosphere densities.

If (1) is replaced by the very similar expression,

$$T(z) = T_{\infty} - (T_{\infty} - T_{120}) \exp[-\sigma \zeta], \quad (2)$$

where  $\zeta$  is the geopotential altitude,

$$\zeta = \frac{(z-120)(R+120)}{R+z}, \quad R = 6356.77 \text{ km}, \quad (3)$$

the diffusive equilibrium equation can be integrated directly to give

$$n(i|z) = n(i|120) \left( \frac{1-a}{1-ae^{-\sigma \zeta}} \right)^{1+\alpha+\gamma} \exp[-\sigma \gamma \zeta], \quad (4)$$

where  $n(i|z)$  is the number density of constituent  $i$  at altitude  $z$ ,

$$a = \frac{T_{\infty} - T_{120}}{T_{\infty}}, \quad (5)$$

$\alpha$  is the thermal diffusion coefficient (equal to  $-0.4$  for  $He$  and zero for  $N_2$ ,  $O_2$  and  $O$ ), and

$$\gamma = \frac{m_i g_{120}}{\sigma k T_{\infty}}, \quad (6)$$

where  $m_i$  is the molecular mass of constituent  $i$ ,  $k$  is Boltzmann's constant, and

$$g_{120} = 980.665 \left( 1 + \frac{120}{R} \right)^{-2} = 944.655 \text{ cm sec}^{-2} \quad (7)$$

is the acceleration of gravity at 120 km. This relationship (4) was derived by Bates (1959; cf. Stein and Walker, 1965) without allowance for thermal diffusion.

If

$$\sigma = s + \frac{1}{R+120} = s + 0.00015, \quad (8)$$

the temperatures given by (1) and (2) are identical at the altitude where

$$s(z-120) = \sigma \zeta = 1 \quad (9)$$

as well as at 120 km and at infinity. The fractional difference between the two temperatures is given approximately by

$$\frac{\Delta T}{T} = \frac{a(z-120)}{(R+120)[e^{s(z-120)} - a]} [1 - s(z-120)] \quad (10)$$

and is less than one per cent at all altitudes for  $T_{\infty} = 200\text{K}$ . The fractional difference is even smaller for smaller values of  $T_{\infty}$ . Thus analytic density profiles given by (4) will differ negligibly from profiles obtained by numerical integration using Jacchia's temperature profile (1). The analytic representation is easier to compute than the representation involving numerical integration.

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